NATURAL CONVECTION OF A DUSTY GAS IN A PLANE CLOSED REGION

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Within the framework of a one-velocity, one-temperature model of the medium, the natural convective motion of a dusty gas is investigated numerically in a plane region of square cross section in the case of side heating.

The problem of gas convection in a region of square cross section with side heat supply has long attracted the attention of research workers from the standpoint of studying the nonstationary characteristics of convection and specific features of the hydrodynamic field. Detailed information on works that use the Boussinesq approximation is given in [1-3]. In [4-6] the Navier-Stokes equations were solved for a compressible gas. At the present time this problem is considered as a standard test to verify various numerical procedures intended for describing natural convective flows [7].

In the present paper we investigate the extension to the case of a disperse medium (a compressible gas with solid particles), which is considered in a one-velocity, one-temperature approximation.

1. Let a plane cavity of square cross section be filled with a dusty gas. The side walls have constant but different temperatures, whereas the lower and upper walls are thermally insulated. In an initially stagnant medium natural convective motion is induced by the force of gravity. The problem is to study the specific features of the convective processes and to elucidate the part played by the dispersed impurity.

Within the confines of the basic assumptions of the mechanics of heterogeneous media [8] the gas and particles are considered as interacting and interpenetrating solid media. The carrier phase is a viscous compressible heat-conducting gas.

The particles are considered to be small enough for the disperse mixture to be considered within the scope of a one-velocity, one-temperature model [8]. The collisions of particles and their diffusion, evaporation, adhesion, and fragmentation are not taken into account.

In dimensionless variables, the equations that describe the flow of a gas suspension have the form

$$\frac{d\rho}{dt} = -\rho\,\nabla\mathrm{U}, \quad P = \rho_1 T, \quad \rho = \rho_1 + \rho_2, \quad \rho\frac{d\mathrm{U}}{dt} = -\frac{1}{\gamma \mathrm{M}^2}\nabla P +$$

$$+\frac{1}{\mathrm{Re}}\left(\nabla\mathrm{U} + \frac{1}{3}\nabla(\nabla\mathrm{U})\right) + \rho\mathbf{g}, \quad (\rho_1 + \gamma_2\rho_2)\frac{dT}{dt} = -(\gamma - 1)\,P\,\nabla\mathrm{U} +$$

$$+\frac{\gamma}{\mathrm{Re\,Pr}}\Delta T, \quad \frac{\partial\rho c}{dt} + \nabla(\rho c\mathrm{U}) = 0, \quad c = \frac{\rho_2}{\rho}, \tag{1}$$

where c is the mass concentration of particles; $\mathbf{g} = (0; -1)$ is the volumetric force acceleration vector.

In changing to dimensionless variables we use the following scales: length L (the side of the region considered), velocity $\sqrt{Lg}$ (g is the free fall acceleration), time $\sqrt{L/g}$, density ρ_{10}, temperature T_0 (ρ_{10} and T_0 are the initial density and temperature of the gas near the cold vertical boundary), pressure $P_0 = R\rho_{10}T_0$ ($R = R_0/\mu$, R_0 is the universal gas constant, μ is the molecular weight of the gas). Equation (1) contains the dimensionless

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complexes $M = \sqrt{Lg/\gamma RT_0}$, $Re = L\sqrt{Lg}\rho_{10}/\eta$, $Pr = c_P\eta/\lambda$, which are the Mach, Reynolds, and Prandtl numbers, $\gamma = c_P/c_V$; $\gamma_2 = c_2/c_P$.

System (1) is solved in the Cartesian rectangular coordinate system, the x axis coincides with the lower boundary of the region, and the y axis coincides with the left vertical boundary. The temperature $1 + T_a$ is maintained on the left wall ($x = 0$) and the temperature 1 on the right; T_a is the dimensionless difference of temperatures. The lower and upper boundaries are thermally insulated: $\partial T/\partial y = 0$ at $y = 0$ and $y = 1$. The no-slip condition ($U = 0$) is fulfilled on all the boundaries.

At the initial instant the linear distribution of temperature in a stagnant medium is assigned: $T(0, x, y) = 1 + T_a(1-x)$, $P(0, x, y) = 1$, $\rho_1(0, x, y) = P/T$, corresponding to the stationary state in the absence of the force of gravity. Initially the concentration of dust is constant within the entire region: $c(0, x, y) = c_0$. Under these initial and boundary conditions the concentration turns out to be independent of time, i.e., the last equation in system (1) has a trivial solution: $c(t, x, y) = c_0$.

In contrast to the popular model of a passive impurity in system (1), the effect of particles on the motion of the gas due to their weight and heat capacity is taken into account. Instead of the density and heat capacity of the gas the system of equations includes the effective parameters. This is noted in [9].

2. System (1) was solved numerically with the aid of an implicit difference scheme [5, 6] on a three-dimensional 21 × 21 grid; the value of the time step τ, interrelated with the space step h, corresponded to the Courant number $Ku = \tau/hM = 4$. The accuracy of the calculations was controlled from the mass balance $\int_0^1\int_0^1 \rho\,dx\,dy$, which was preserved within 1%. The following values of the dimensionless parameters were used: $M = 0.1$; $Re = 250$; $Pr = 0.7$; $\gamma = 1.4$; $T_a = 0.5$; $\gamma_2 = 0$-10; $c_0 = 0$-0.7. At $\gamma_2 = 0$, $c_0 = 0$ the problem is reduced to the study of pure gas convection and coincides with that investigated in [5, 6]*. Calculations were made on an ES-1055M electronic computer; each version required about 40 minutes of machine time.

3. The force of gravity induces convective circulational motion in a gas with an ascending flow near a hot wall and a descending flow near a cold wall. The introduction of impurity into the gas changes the properties of the medium. Two additional parameters appear: c_0, which is proportional to the total mass of the particles, and γ_2, which is proportional to their heat capacity.

To reveal the influence of each parameter on the process dynamics, we will analyze the results of calculations of the following limiting regimes. In the first one, the impurity is absent, $\rho_2 = \rho c_0 = 0$. In the second one, particles are introduced with the parameters $c_0 = 0.4$, $\gamma_2 = 2.0$. Two other versions are purely hypothetical, namely, the particles have heat capacity with $\gamma_2 = 2.0$ but do not have weight (in the continuity and momentum equations of system (1) the medium density ρ was replaced by the gas density ρ_1) or, conversely, they have weight but do not contribute to the heat capacity: $c_0 = 0.4$, $\gamma_2 = 0$.

In Fig. 1 the change in the mean Nusselt number on the hot wall

$$Nu = -1/T_a \int_0^1 \frac{\partial T}{\partial x}\,dy$$

with time is shown for all of the regimes indicated. Curve 1, corresponding to pure gas convection, completely coincides with predictions [5, 6]. Account for one of the factors, for example, just the thermal (curve 3) or just the weight (curve 4) factor, leads to a heat flux from the hot wall to the gas that exceeds the value for a pure gas (curve 1). Since the effect of both factors leads to enhancement of heat transfer, Nu attains the greatest values on introduction of particles (curve 2).

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* Within this very limit, we made a comparison with the characteristics of a stationary flow investigated in the Boussinesq approximation [7]. In particular, we obtained that at $Pr = 0.71$ and $Ra = 10^4$ (Ra is the Rayleigh number expressed in terms of the parameters used as $Ra = Pr \cdot Gr = Pr \cdot Re^2 \cdot T_a$ (see Eq. (5)), the computed value for the Nusselt number on the left boundary $Nu = 2.10$ corresponds to the test value $Nu^* = 2.26$ [7], accurate to 8%.

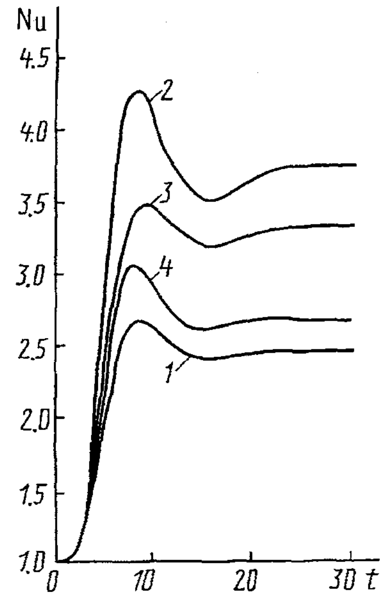

Fig. 1. Nusselt number Nu on a hot vertical boundary vs time t for $c_0 = 0$, $\gamma_2 = 0$ (1), for $c_0 = 0.4$, $\gamma_2 = 2.0$ (2), in the case of the thermal effect of particles with $\gamma_2 = 2.0$ ($c_0 = 0.4$) (3) and the weight effect of particles with $c_0 = 0.4$, $\gamma_2 = 0$ (4).

The mechanism of the influence of particles on heat transfer follows from the analysis of the internal structure of developed flow. In Fig. 2a the spatial distribution of the vertical velocity component $v(x; 0.5)$ is given in the central horizontal section at time $t = 30$, when the motion becomes stationary. It is seen that the "inclusion" of the weight effect of the particles slightly changes the form of the velocity profile but substantially increases the velocity scale (compare curves 1 and 4). This is caused by the increase in the Archimedian force in a dusty medium as compared with a pure gas because of the greater density of the medium; the difference between the densities of the medium near the cold and hot walls increases by $1/(1-c_0)$ times ($\rho = \rho_1/(1-c_0)$). The intensification of flow leads, in turn, to the enhancement of convective heat transfer.

Let us consider the thermal effect of particles. The effective heat capacity of a gas suspension is higher than that of a pure gas. Therefore, a mixture ascending along a hot wall is heated up more slowly, whereas that descending near a cold boundary cools off more slowly. This is seen from Fig. 2b, in which the temperature distribution $T(x; 0.5)$ in the central section is given. As the effective heat capacity of the medium rises, the temperature curves near the boundaries become steeper (compare curves 1 and 3), the temperature gradients increase, and, accordingly, the Nu number, which is determined by them, grows (Fig. 1). Because of the increase in the temperature inhomogeneity near the walls, the density in the boundary zones changes more sharply. The dynamic boundary layers become narrower (Fig. 2a), the main motion occurs near the vertical boundaries, and there is virtually a stagnant mixture at the center.

Thus, account for the heat capacity of particles leads, first, to enhancement of convective heat transfer and, second, to diminution of the vertical boundary layers and enlargement of the stationary core at the center.

As the concentration of the impurity grows, the influence of heat capacity on the gas increases. With rather large additives $c_0 \simeq 0.7$ the dust intensifies the occurring processes so strongly that rearrangement in the internal flow structure occurs. Similar changes are observed upon increase in γ_2. For a dusty medium with a high concentration of impurity ($c_0 = 0.7$, $\gamma_2 = 2.0$, see Fig. 3b) or with an increased heat capacity of the particles ($c_0 = 0.2$, $\gamma_2 = 10.0$, see Fig. 3c) secondary structures develop inside of the flow, which is not observed in a pure gas (Fig. 3a). The nature of their formation is the same. As c_0 or γ_2 increases, the horizontal temperature differences near the side boundaries grow. Even at a small distance from the hot wall the gas suspension layers are rather cold. Ascending upward and moving along the upper boundary, the mixture cools off rapidly, and its density increases. The heavy mixture sinks by gravity, not reaching the right vertical boundary. Vortical motion is formed

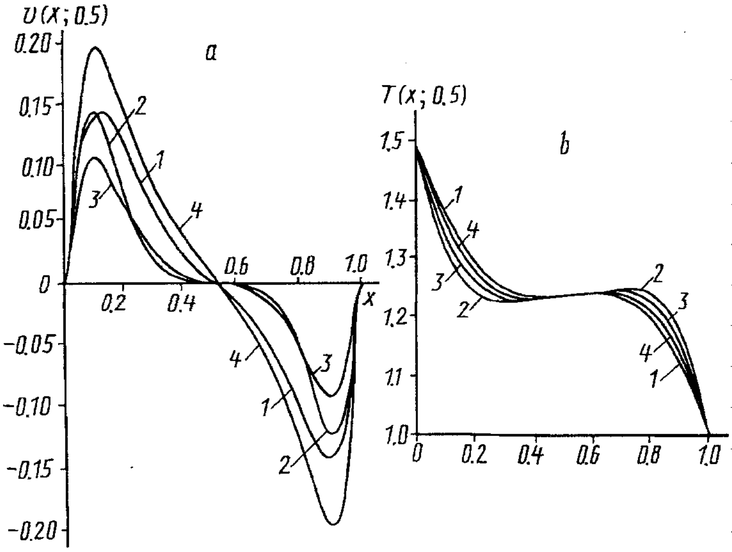

Fig. 2. Distribution of the vertical velocity component $v(x, 0.5)$ (a) and the temperature $T(x, 0.5)$ (b) in the central horizontal section at time $t = 30$ for $c_0 = 0$, $\gamma_2 = 0$ (1), for $c_0 = 0.4$, $\gamma_2 = 2.0$ (2), in the case of the thermal effect of particles with $\gamma_2 = 2.0$ ($c_0 = 0.4$) (3) and the weight effect of particles with $c_0 = 0.4$, $\gamma_2 = 0$ (4).

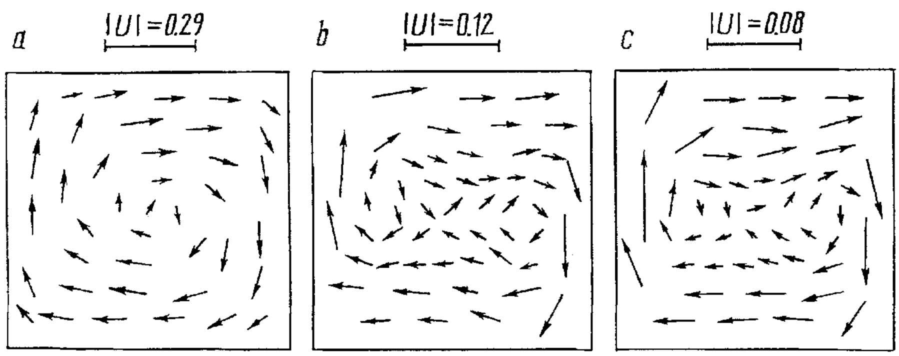

Fig. 3. Velocity field at time $t = 30$ for $c_0 = 0$, $\gamma_2 = 0$ (a), $c_0 = 0.7$, $\gamma_2 = 2.0$ (b) and $c_0 = 0.2$, $\gamma_2 = 10.0$ (c).

near the left-hand hot wall. A similar process occurs near the right-hand side boundary. The gas suspension that moves to the left-hand boundary is heated rapidly, expands, and, not reaching the boundary, ascends upward under the action of buoyancy and forms a vortex near the right-hand cold boundary. A similar two-vortex structure was discovered in [10] in a pure gas (Pr = 0.7) when the Grashof number increased to Gr $\simeq 10^5$.

4. The laws governing dusty gas convection are characterized by modified similarity numbers that take into account the presence of impurity. We will obtain their expressions on the basis of the Boussinesq approximation.

Assuming in the system of equations for a gas suspension (1), written in dimensional form, that the temperature and pressure deviate by a small amount T' and P' from their mean values and taking the compressibility of the medium into account only in the lift force, we obtain

$$\nabla \mathrm{U} = 0\,, \quad \frac{d\mathrm{U}}{dt} = -\frac{1}{\rho_0}\nabla P' + \frac{\eta}{\rho_0}\nabla \mathrm{U}' - \beta \mathbf{g} T'\,, \quad \rho_0 = \rho_{10} + \rho_{20}\,,$$

$$\frac{dT}{dt} = \frac{1}{1 + c_0(\gamma_2 - 1)} \frac{\lambda}{\rho_0 c_P} \Delta T'\,, \tag{2}$$

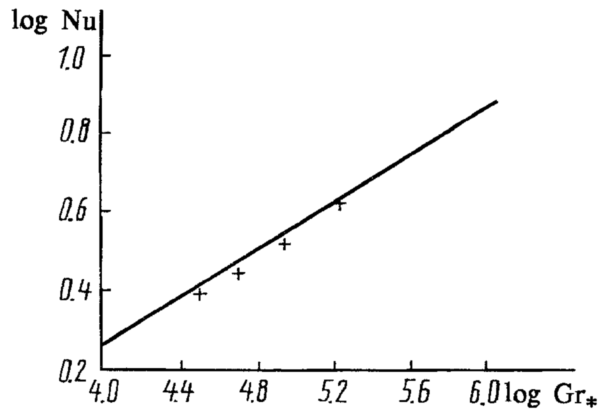

Fig. 4. A plot of the Nusselt number Nu on a hot boundary vs the Grashof number Gr_* in a pure gas constructed from the expression $\mathrm{Nu} = 0.12\mathrm{Gr}_*^{0.3}$ given in [4]. The results of the present authors for a gas suspension with $\gamma_2 = 1.0$ and $c_0 = 0.0, 0.2, 0.4, 0.6$ are denoted by dots.

where ρ_0 is the medium density at equilibrium; $\beta = -1/\rho_0(\partial\rho/\partial T)_P$ is the thermal expansion coefficient of the medium.

We introduce the effective coefficients of kinematic viscosity ν_* and thermal diffusivity κ_* in the following way:

$$\nu_* = \frac{\eta}{\rho_0} = (1 - c_0)\,\nu\,, \quad \nu = \frac{\eta}{\rho_{10}}\,,$$

$$\kappa_* = \frac{1}{1 + c_0(\gamma_2 - 1)}\,\frac{\lambda}{\rho_0\, c_P} = \frac{1 - c_0}{1 + c_0(\gamma_2 - 1)}\,\kappa\,, \quad \kappa = \frac{\lambda}{\rho_{10}\, c_P}\,, \tag{3}$$

where ν and κ are the corresponding parameters of a pure gas. The use of ν_* and κ_* reduces Eqs. (2) to the form of the gas equations, and the modified similarity numbers Pr_* and Gr_*, constructed on their basis, permit one to consider the dusty medium as a gas with these effective parameters:

$$\mathrm{Pr}_* = \frac{\nu_*}{\kappa_*} = (1 + c_0(\gamma_2 - 1))\,\mathrm{Pr}\,, \quad \mathrm{Gr}_* = \frac{g\beta T_a L^3}{\nu_*^2} = \frac{1}{(1 - c_0)^2}\,\mathrm{Gr}\,. \tag{4}$$

The numbers Pr and Gr characterize the gas phase; in the case of a perfect gas

$$\mathrm{Gr} = T_a \cdot \mathrm{Re}^2 \tag{5}$$

and for the conditions selected $\mathrm{Gr} = 3.1\cdot 10^4$.

In Fig. 4 the function $\mathrm{Nu}(\mathrm{Gr}_*)$ is given, which is plotted for a pure gas from the expression $\mathrm{Nu} = 0.12\mathrm{Gr}_*^{0.3}$ for $\mathrm{Pr} = 0.7$ [4]. Here Gr_* corresponds to a pure gas. For comparison, computations were made for a homogeneous gas suspension with $\gamma_2 = 1.0$ and $c_0 = 0.0, 0.2, 0.4, 0.6$; the remaining parameters are the previous ones (for the value of γ_2 selected it follows from Eq. (4) that $\mathrm{Pr}_* = \mathrm{Pr}$). The predicted points in Fig. 4 lie rather close to the curve for a pure gas, testifying to the possibility of representing a disperse medium as a gas with altered properties in the considered range of parameters.

The effective parameters also characterize the formation of secondary vortices in a dusty medium (see Fig. 3). For a medium with $c_0 = 0.7$, $\gamma_2 = 2.0$ (regime of Fig. 3b), starting from Eq. (4) we obtain that $\mathrm{Gr}_* = 3.5\cdot 10^5$ and $\mathrm{Pr} = 1.2$. This agrees with the transient values in a pure gas [10]. The flow of a gas suspension with $c_0 = 0.2$ and $\gamma_2 = 10.0$ (Fig. 3c) is described by the values $\mathrm{Gr}_* = 4.9\cdot 10^4$ and $\mathrm{Pr} = 2.6$.

We note that the obtained analogy between the convection of a dusty medium and of a pure gas with modified properties is valid only for a one-velocity, one-temperature mixture with a constant concentration of dust.

Let us evaluate the dimensions of particles for which the model used here is applicable. Within the scope of the Stokes law of friction the characteristic time of the velocity relaxation of particles is determined by the formula $\tau_u = \rho_2^0 d^2/18\eta$ [8], and the time of their temperature relaxation by $\tau_T = \rho_2^0 d^2 c_2/12\lambda$ [8], where ρ_2^0, d_0 are the true density and the diameter of a particle. When a one-velocity, one-temperature model is used, the values of τ_u and τ_T should be much smaller than the characteristic time of the problem $\sqrt{L/g}$. Let us estimate the value of d_0 by τ_u, since τ_u and τ_T are close in magnitude. After some transformations, the condition $\rho_2^0 d^2/18\eta \ll \sqrt{L/g}$ yields $d_0 \ll \sqrt{18\varepsilon/\mathrm{Re}L}$, where $\varepsilon = \rho_{10}/\rho_2^0$ is the ratio of the true densities of the phases. For a region of size $L = 10^{-2}$ m for $\varepsilon = 10^{-3}$ and Re = 250 we obtain that $d_0 \ll 10^{-4}$ m. A coarser impurity in which the velocity and temperature inertias of the particles as well as their their deposition are substantial should be considered within the framework of the two-velocity, two-temperature model.

Thus, the introduction of a fine disperse impurity into a gas leads to the intensification of convective heat transfer and to the thinning of the boundary layers near the side boundaries. At a sufficiently large concentration or heat capacity of the dust a convective vortex can break into two vortices. Within the scope of the considered one-velocity, one-temperature model of the medium with a constant concentration of impurity, we obtained an analogy between the convection of a dusty gas and of a pure gas with modified similarity numbers.

NOTATION

ρ_1, P, mean density and pressure of the gas; ρ_2, mean density of the impurity; ρ, $\mathbf{U} = (u, v)$, T, density, velocity, and temperature of the mixture; t, time; x, y, Cartesian coordinates; g, free fall acceleration; L, side of the square region; M, Re, Pr, Gr, Mach, Reynolds, Prandtl, and Grashof numbers; Pr_*,Gr_*, modified Prandtl and Grashof numbers; Nu, Nusselt number; T_a, difference of temperatures on the side boundaries of the region; $\gamma = c_P/c_V$ where c_P and c_V are the heat capacities of the gas at constant pressure and volume; $\gamma_2 = c_2/c_P$, where c_2 is the heat capacity of the impurity substance; c_0, impurity concentration; η, ν, λ, κ, coefficients of dynamic and kinematic viscosity, thermal conductivity, and thermal diffusivity of the gas; ν_*, κ_*, effective coefficients for the mixture as a whole; d_0, diameter of the particles, m.

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